# SET THEORY

A set is a well-defined collection of objects, known as elements or members of the set. Sets are usually denoted by capital letters & elements are usually denoted by small letters. If 'a' is an element of a set A, then we write a ϵ A (a belongs to A) otherwise a ∉ A (a doesn't belong to A).

Representation of sets:

1. **Tabular or Roster form** → In this form, elements are listed within the pair of brackets { } and are separated by commas.
   e.g.— N = {1, 2, 3, 4 ....} is a set of natural numbers
2. **Set-builder or Rule form**: In this form, set is describe by a property that its member must satisfy.
   e.g. — A = {x : x is natural number less than 10}
3. **Statement form:** In this representation, well defined description of the elements of the set is given.
   e.g. — The set of all even numbers less than 10.

Different Types of sets:

1. **Null set** → A set which does not contain any element is called a null set or an empty set or a void set.
2. **Singleton set** → A set which contain only one element.
3. **Finite set** → A set is called a finite set, if it is either void or its elements can be counted
   The number of distinct elements of a finite set A is called the cardinal number & it is denoted by n(A).
4. **Infinite set** → A set which has unlimited number of elements is called infinite set.
5. **Equivalence sets:** Two finite sets A and B are equivalent, if their cardinal numbers are same.
6. **Equal sets:** Two sets are said to be equal if both have same elements.
   Note:– Equal sets are equivalent but equivalent sets may or may not be equal.
7. **Subset:** If every element of set A is an element of set B, then A is called a subset of B sit is denoted by A ⊆ B.
8. **Superset** → If set B contains all elements of set A, then B is called superset of A & it is denoted by B ⊇ A.
9. **Proper subset** → A set A is said to be a proper subset of set B, if A is a subset of B & A is not equal to B. It is denoted by A ⊂ B.
10. **Universal set** → Universal set is a set which contains all objects, including itself. It is denoted by U.
11. **Power set** → The set of all the possible subsets of A is called the power set & is denoted by P (A).

   **Note:-**
   1. The total number of subsets of a finite set containing n elements is $2^n$.
   2. The total number of proper subsets of a finite set containing n elements is ($2^n$ –1).
   3. If a set A has n elements, then its power set will contain $2^n$ elements.

Operations on sets:

1. **Union of two sets:** The union of two sets A and B is the set of elements which are in A, in B or in both A & B. The union of A & B is denoted by A ∪ B.
2. **Intersection of two sets:** The intersection of A & B is the set of all those elements which belong to both A & B & is denoted by A ∩ B.
3. **Disjoint of two sets:** Two sets A & B are said to be disjoint if they don't have any common element (i.e. A ∩ B = ϕ).

4. **Difference of two sets:** The difference of sets A & B is the set of all those elements of A which do not belong to B. & is denoted by $(A - B)$ or $A \setminus B$.
5. **Symmetric difference of two sets** : The symmetric difference of sets A & B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.
6. **Complement of a set:** The complement of a set A is the set of all those elements which are in universal set but not in A. It is denoted by $A^1$ or $A^C$ or $U - A$.

<u>Laws of Algebra of sets:</u>

1. (a) $A \subseteq A \ \forall \ A$

   (b) $\phi \subseteq A \ \forall \ A$

   (c) $A \subseteq U, \forall \ A$ in $\cup$

   (d) $A = B \Leftrightarrow A \subseteq B, B \subseteq A$.
2. **Idempotent laws:**
   (a) $A \cup A = A$
   (b) $A \cap A = A$
3. **Identity laws:**
   (a) $A \cup \phi = A$
   (b) $A \cap \phi = \phi$
   (c) $A \cap U = A$
   (d) $A \cup U = U$
4. **Commutative law**
   (a) $A \cup B = B \cup A$
   (b) $A \cap B = B \cap A$
5. **Associative laws**
   (a) $(A \cup B) \cup C = A \cup (B \cup C)$
   (b) $(A \cap (B \cap C) = (A \cap B) \cap C$
6. **Distributive law**
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
7. **De-Morgan's law**
   (a) $(A \cup B)' = A' \cap B'$
   (b) $(A \cap B)' = A' \cup B'$
8. (a) $A - (B \cup C) = (A - B) \cap (A - C)$
   (b) $A - (B \cap C) = (A - B) \cup (A - C)$
   (c) $A - B = A \cap B' = B' - A'$
   (d) $A - (A - B) = A \cap B$
   (e) $A - B = B - A \Leftrightarrow A = B$
   (f) $A \cup B = A \cap B \Leftrightarrow A = B$
   (g) $A \cup A' = U$
   (h) $A \cap A' = \phi$

**Important results:**

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

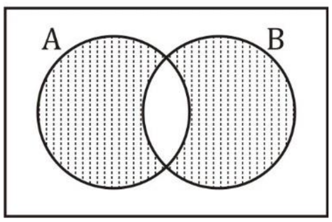
4. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
5. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$.
6. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$.
7. $n(A') = n(U) - n(A)$.
8. $n(A \cap B') = n(A) - n(A \cap B)$.

**EXAMPLES**

1. The number of proper subsets of the set {1, 2, 4, 6} is:

Sol. Number of proper subsets of the set {1, 2, 4, 6} = $2^4 - 1 = 16 - 1 = 15$.

2. If $n(A) = 4$, $n(B) = 4$, $n(A \cap B) = 2$, then the area of shaded portion is —




Sol. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$.
$= 4 + 4 - 4$
$= 4$

3. In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak English. What is the number of students who can speak Hindi only ?

Sol. $n(H) = 475$, $n(E) = 200$, $n(H \cup E) = 500$
$n(H \cap E) = n(H) + n(E) - n(H \cup E)$
$= 475 + 200 - 500 = 175$.
Number of students who can speak Hindi only
$= n(H) - n(H \cap E)$
$= 475 - 175 = 300$

4. Let $n(U) = 800$, $n(A) = 250$, $n(B) = 300$, $n(A \cap B) = 350$ then $n(A' \cap B')$ is equal to

Sol. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 250 + 300 - 350$
$= 200$
$n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
$= 800 - 200$
$= 600$

5. Let $U = \{x \in N : 1 \le x \le 8\}$ be the universal set, N being the set of natural numbers. If A = {1, 2, 3, 4} and B = {2, 4, 6, 8}. Then what is the complement of (A – B) ?

Sol. A = {1, 2, 3, 4) B = {2, 4, 6, 8)
(A – B) = {1, 3}
(A – B)′ = U – (A – B)
= {1, 2, 3, 4, 5, 6, 7, 8} – {1, 3}
= {2, 4, 5, 6, 7, 8}

# IMPORTANT QUESTIONS

Q1. If A = {x : x is an even natural number},
B = {x : x is a natural number and multiple of 5} and
C = {x : x is a natural number and multiple of 10}, then what is the value of $A \cap (B \cup C)$?
(a) {10, 20, 30, …}
(b) {5, 10, 15, 20…….}
(c) {2, 4, 6……..}
(d) {20, 40, 60, ……….}

Q2. If a set a Contains 60 elements and another set B contains 70 elements and there are 50 elements in common, then how many elements does $A \cup B$ contain?
(a) 130
(b) 100
(c) 80
(d) 70

Q3. Let $x \in \{2, 3, 4)$ and $y \in \{4, 6, 9, 10\}$. If A be the set of all order pairs (x, y) such that x is a factor of y. Then, how many elements does the set A contain?
(a) 12
(b) 10
(c) 7
(d) 6

Q4. Which one of the following is a null set?
(a) A = {x is a real number : $x > 1$ and $x < 1$}
(b) B = { x : $x + 3 = 3$}
(c) C = {$\phi$}
(d) D = {x is a real number : $x \geq 1$ and $x \leq 1$}

Q5. In a school there are 30 teachers who teach mathematics or Physics. Of these teachers, 20 teach Mathematics and 15 teach physics, 5 teach both mathematics and physics. The number of teachers teaching only mathematics is
(a) 5
(b) 10
(c) 15
(d) 20

Q6. If A = {x : x is an odd integer} and B = {x : $x^2$ - 8x + 15 =0}.
Then, which one of the following is correct?
(a) A = B
(b) $A \subseteq B$
(c) $B \subseteq A$
(d) $A \subseteq B^C$

Q7. Consider the following in respect of the sets A and B.
I. ($A \cap B \subseteq A$)
II. $(A \cap B) \subseteq B$
III. $A \subseteq (A \cup B)$
Which of the above are correct?
(a) I and II
(b) II and III
(c) I and III
(d) I, II and III

Q8. The set of natural numbers is closed under
I. addition
II. subtraction
III. multiplication
iv. division
which of the above is/are correct?
(a) only I
(b) both I and III
(c) I, II and III
(d) both III and IV

Q9. In a class of 110, students, x students take both mathematics and, 2x +20 students take statistics. There are no students who take neither mathematics nor statics. What is x equal to?
(a) 15
(b) 20
(c) 25
(d) 30

Q10. Out of 105 students taking an examination English and mathematics 80 students pass in English 75 students pass in mathematics 10 students fail in both the subjects. How many students fail in only one subject?
(a) 26
(b) 30
(c) 35
(d) 45

## Solutions

**S1. Ans.(a)**

**Sol.** We know that

$A \cap (B \cup C) = \{A \cap B) \cup (A \cap C)$

Example

A = set of an even natural number

A = {2, 4, 6, 8, 10, 12……..}

B = Set of natural number and multiples of 5.

B = {5, 10, 15, 20, 25…..}

C = Set of natural number and multiple of 10

C = {10, 20, 30, 40, 50…..}

$A \cap B = \{2, 4, 6, 8, 10, 12 \ldots\}$

$\cap \{5, 10, 15, 20, 25, \ldots\}$

$= \{10, 20, 30, \ldots\}$

$A \cap C = \{2, 4, 6, 8, 10, 12, \ldots\} \cap \{10, 20, 30, 40, 50\}$

$= \{10, 20, 30, 40, \ldots\}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= \{10, 20, 30, \ldots\} \cup \{10, 20, 30, 40, \ldots\}$

$= \{10, 20, 30, 40, \ldots\}$

**S2. Ans.(c)**

**Sol.**

Here, $n(A) = 60$, $n(B) = 70$, $n(A \cap B) = 50$ and $n(A \cup B) = ?$

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 60 + 70 - 50$

$= 130 - 50 = 80$

**S3. Ans.(d)**

**Sol.** Given that

$x \in \{2, 3, 4\}$

and

$y \in \{4, 6, 9, 10\}$

$A = x \times y$

But, A is set of pairs in which 1$^{st}$ number is factor of second number.

$A = \{2, 3, 4\} \times \{4, 6, 9, 10\}$

$= \{2, 4\}; (2, 6); (2, 10); (3, 6); (3, 9); (4, 4)\}$

Total number of elements = 6

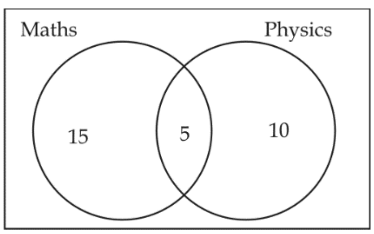
**S4. Ans.(a)**

**Sol.** From option (a),

A = {x is a real number : x > 1 and x < 1}

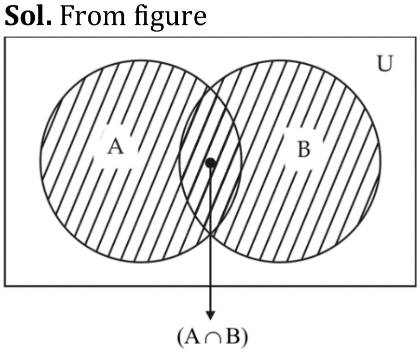
So, there is no element which is greater or less than 1.

So, A is a null set
From option (b)
B = {x : x + 3 = 3} = {0}
= singleton set
From option (c)
C = {ϕ} = Singleton set
From option (d),
D {x is a real number : x ≥ 1 and x ≤ 1}
={1} = singleton set

**S5. Ans.(c)**
**Sol.**
Total number of teachers = 30


Number of teachers who teaches only math = 20 – 15 = 15

**S6. Ans.(c)**
**Sol.** Given that,
A = {x : x is an odd integer}
And, B = {x : $x^2$ - 8x + 15 = 0} = {x : $x^2$ - 5x – 3x + 15 = 0}
= {x : x (x – 5) – 3 (x – 5) = 0}
= {x : (x -5) (x – 3) = 0} = {3, 5}
Since, B has the odd elements,
∴ B ⊆ A

**S7. Ans.(d)**
**Sol.** From figure


$(A \cap B) \subseteq A$ (True)
(A ∩ B) ⊆ B (True)
And, A ⊆ (A ∪ B) Also (True)

Thus, all three statements are correct
Shaded region = $(A \cup B)$

**S8. Ans.(b)**
**Sol.** Set of natural numbers,
N = {1, 2, 3, 4, 5, 6, ………..
(i) Addition $2 + 3 = 5$ is also an element of N/
(ii) Subtraction $2 - 3 = -1 \notin N$
(iii) multiplication $2 \times 3 = 6 \in N$
(iv) Division $\frac{3}{2} = 1.5 \notin N$ (since, N contains only positive integers)
Therefore, the set of natural numbers is closed under addition and multiplication

**S9. Ans.(b)**
**Sol.**
$n(M) = 2x + 20$
$n(S) = 2x + 30$
$n(M \cap S) = x$
$n(M \cup S) = 110$
we know that,
$n(M \cup S) = n(M) + n(S) - n(M \cap S)$
$\Rightarrow 110 = 2x + 20 + 2x + 30 - x$
$\Rightarrow 110 = 3x + 50$
$\Rightarrow 3x = 60$
$\therefore x = 20$

**S10. Ans.(d)**
**Sol.**
Number of students failing in mathematics
$= 105 - 75 = 30$
Number of students failing in English $= 105 - 80 = 25$
$\therefore$ Number of students failing in 1 subject $= (25 + 30) - 10 = 45$